* Definition of a Vector
  + A vector is comprised of (1) a number (magnitude) WITH (2) a direction.
  + Examples:

3 km east 145 mph west

13 steps forward 67 paces sideways

9000 N up 2.5 g towards earth

* + A vector ALWAYS has a magnitude (*with units*) and a direction that defines it. A “scalar” quantity only includes magnitude (*with units*).

[**https://screencast-o-matic.com/watch/cYhtbQpq0c**](https://screencast-o-matic.com/watch/cYhtbQpq0c) **What is a Vector? "Despicable Me" (0:39)**

* Vector quantities
  + A vector quantity is simply a measurement that is represented by a vector. While much of physics deals with vector quantities, there are also non-vector quantities.
  + For some vectors there is an associated non-vector quantity that describes only the amount without a direction.
  + These are some vector quantities and the associated non-vector quantity if one exists:

|  |  |
| --- | --- |
| **Vector Quantity** | **Non-Vector Quantity** |
| Displacement | Distance |
| Velocity | Speed |
| Acceleration |  |
| Force |  |

* + - Generally speaking, “speed” is the same as “velocity.”
    - Technically speaking, however, (*in the science world*) “speed” is only the magnitude (*with units*) while velocity is defined as “speed” plus direction.
    - For example, 35 mph represents speed, while 35 mph **east** represents velocity.
    - Vector Quantities can be added (or subtracted) graphically and/or mathematically.
* Graphically Representing a Vector
  + Vectors are indicated by lines with arrows.
  + Vectors are often drawn to scale and the magnitude can be measured just like a scale on a map.
  + A vector’s magnitude will be indicated by a number written alongside the vector.

15 meters

36 N

* + Notice that these vectors have the TWO key parts: magnitude and direction.
    - The magnitude is indicated by number written (*which may be scaled down to fit the page*), while the direction is indicated by the arrow.
* Vector Addition (and Subtraction) in One Dimension
  + To add vectors, put the vectors “head-to-tail” or “tail to head”  put the tail of one vector at the head of the other vector (*as shown below*).

100 N

100 N

200 N

300 N

200 N

Adding the Vectors Sum of the Vectors

* + - The sum of the vectors is called the resultant. The resultant has the same effect as the original vectors.
    - The resultant is the single vector that could represent the sum of several vectors.
  + When adding (or subtracting) vectors, the order of addition (or subtraction) does not matter. In the example above, one may add them in more than one way.

100 N

200 N

300 N

Adding the Vectors Resultant

* To add vectors in opposite directions [*subtraction of vectors*], subtract the smaller vector magnitude from the larger vector magnitude to get the magnitude and keep the direction of the resultant vector.

100 N

100 N

200 N

100 N

200 N

opposite directions

Resultant

* Adding/Subtracting Vectors in More Than One Dimension
  + - Vector Quantities are independent of each other. [Technically this applies to perpendicular vectors]
      * For instance, a boat traveling 9.4 m/s at 32 north of east can also be described as traveling both east at 8.0 m/s and north at 5.0 m/s at the same time.
      * The boat traveling east does not change the velocity of the boat traveling north.

8.0 m/s

5.0 m/s

9.4 m/s

32°

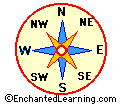
* + Realistically, most vectors are not “due north, south, east or west” and do not travel in the same or opposite direction.
    - For instance, what if we have a vector that heads due east and a second vector that heads 45° southeast? We can graphically work with these vectors:

Due East

45° SE

* It is helpful to know “Cardinal Directions” and “Intercardinal Directions” to help orient oneself. [*see next page*]

Cardinal Directions Intercardinal Directions



**N**

**E**

**S**

**W**

NNE

SNE

NSE

SSE

NNW

SNW

NSW

SSW

* Using the two vectors (*one running due east and the other* 45° *SE*), we can graphically add the two vectors “head to tail” to find the resultant.

Resultant

* + When adding (or subtracting) vectors, the order of addition (or subtraction) does not matter. In the example above, *the dashed line represents the sum of the two vectors*. Notice, one may add them in more than one way.

R

Example 2: Notice, one may add them in more than one way and still get the same **resultant** [R].

R

R

Find the resultant of the following three vectors:

a

b

c

resultant

a

b

c

* “a” is the **equilibrant** (equal and opposite to the resultant.

a

b

R

c

* Remember, one can add these vectors in more than one way to find the resultant. The resultant magnitude and direction will be the same.
* Be sure to “add” the vectors “head to tail”
* Graphic representation is very helpful, but not as accurate as using mathematical computation. This requires trigonometry (*because angles are involved*).
* Vector Analysis Using Components

* Normally, vector analysis uses components based on right triangles.
* This allows one to utilize simple trigonometric and geometric functions to resolve vectors.
  + Using angle θ, the following trig functions apply:

Hypotenuse (R)

Adjacent side (x)

Opposite side (y)

θ

Sin θ = opposite / hypotenuse

Cos θ = adjacent / hypotenuse

Tan θ = opposite / adjacent

* To find sides of a right triangle, one should use the Pythagorean theorem

R2 = x2 + y2

* Vector Resolution  The process of finding the magnitude of a component in a given direction is called vector resolution.
  + - Magnitudes used can be resolved into its vertical and horizontal components. This is dealing with perpendicular components.

Example: A wind with a velocity of 40.0 km/h blows at an angle of 30.0° as shown. What is the component of the wind’s velocity toward 90°? What is the component of the wind’s velocity toward 0°?

* The vector to the left represents the wind velocity Vw

Vw

30.0°

* To resolve this vector, one should resolve the vector into its x & y components.
* The x-component velocity is represented by Vx

Vx

Vy

30.0°

Vw

* The y-component is represented by Vy

Trigonometrically, one can determine the components as follows:

**y component** Sin 30.0° = opp / hyp  Sin 30.0° = **Vy** / Vw  **Vy** = Vw Sin 30.0°

Therefore, **Vy** = (40.0 km/h)(0.5) = 20.0 km/h

**x component** Cos 30.0° = adj / hyp  Cos 30.0° = **Vx** / Vw  **Vx** = Vw Cos 30.0°

Therefore, **Vx** = (40.0 km/h)(0.866) = 34.6 km/h

For a given right triangle problem, one can find the x and y components as follows:

θ

X

Y

R

**x component** **Xx** = R Cos θ

**y component** **Yy** = R Sin θ

* Vector Resolution using NON-Right Triangles

A. Using Components

1. When working with vectors that do not easily form right triangles, one can add extra steps to resolve the resultant vector using the x & y components.
2. Often students will use this method rather than face the more intimidating “Law of Cosines” or “Law of Sines”

Example Two boys needed to move a heavy box that possessed a “eye” ring at one end. They decided to place the rope through the eye ring so that both could pull on the rope simultaneously. One of the boys pulled due east with a force of 45 newtons and the other pulled southeast with a force of 65 newtons. What is the resultant direction of their combined forces?

45 N

65 N

45°

Notice that this will not form a right triangle.

Redraw the vectors “head to tail” to see the resultant

45 N

65 N

R

45°

To use components in this problem, one would have to extend the vectors (*as shown*) and resolve the components.

x component of force (Fx)

45 N

65 N

R

45°

Fy

Notice the components to resolve

for “R” have changed.

y component of force

Using geometry, we can label some angles:

45 N

65 N

R

45°

45°

Fy

Fx

Since we now have a right triangle

we can label another angle

Find the Fx and Fy of the new triangle.

**x component** **Fx** = R Cos θ Fx = (65 N) cos 45° = 46 N

**y component** **Fy** = R Sin θ Fy = (65 N) cos 45° = 46 N

45 N

65 N

R

45°

45°

46 N

46 N

91 N

Use the Pythagorean

Theorem to calculate “R”

R2 = x2 + y2 therefore,

R = √(x2 + y2)

R = √(912 + 462) = √(8281 + 2116) = √10397

**R = 102 N**

B. Using the Law of Cosines

* + The typical trigonometric functions of sine, cosine, tangent, etc. only work with vectors that can form right triangles.
  + To deal with vectors that do not form right triangles, one must use either the Law of Cosines or the Law of Sines. For this class, we will not use the Law of Sines.
  + Consider the triangle below, having sides A, B, C and corresponding angles a, b, c.

c

A

B

C

a

b

* + Notice that line C represents the resultant of lines A + B.
  + One can mathematically determine the resultant if the two sides (A & B) are known and the angle (c) between them.
  + The law of cosines states that C2 = A2 + B2 – 2ABcos(c).

Example Two boys needed to move a heavy box that possessed a “eye” ring at one end. They decided to place the rope through the eye ring so that both could pull on the rope simultaneously. One of the boys pulled due east with a force of 45 newtons and the other pulled southeast with a force of 65 newtons. What is the resultant direction of their combined forces?

45 N

65 N

45°

Notice that this will not form a right triangle.

Redraw the vectors “head to tail” to see the resultant

45 N

65 N

R

45°

Since supplementary angles add up to 180,

the interior angle opposite “R”  180° - 45° = 135°.

Use the Law of Cosines to determine “R”

45 N

65 N

R

135°

R2 = A2 + B2 – 2ABcos(135°)

R2 = 452 + 652 – 2(45)(65)cos(135°)

R = √(452 + 652 – 2(45)(65)cos(135°))

R = √(2025 + 4225 – 5850(-0.0707))

**R** = √(6250 + 4137)) = √(10387) = **102 N**

Notice, that this is the same answer we obtained using components, but took less steps.

You need to choose which method works best for you.