Chapter 9

Circular Motion

The linear speed of the hamster is related to the rotational speed of the track.

Which moves faster on a merry-go-round, a horse near the outside rail or one near the inside rail? Why don't the riders fall out of the rotating carnival ride in Figure 9.1 when the rotating platform is raised? If you swing a tin can at the end of a string in a circle over your head and the string breaks, does the can fly directly outward or does it continue its motion without changing its direction? Why do astronauts orbiting in a space shuttle float in a weightless condition, whereas astronauts orbiting in some future rotating space station will experience normal Earth gravity? These questions indicate the flavor of this chapter. We begin by discussing the difference between rotation and revolution.

Figure 9.1 ▶
Why do the occupants of this carnival ride not fall out when it is tipped almost vertically?

9.1 Rotation and Revolution

Both the rotating platform of the carnival ride shown in Figure 9.1 and an ice skater doing a pirouette turn around an **axis.** An axis is the straight line around which rotation takes place. When an object turns about an internal axis—that is, an axis located within the body of the object—the motion is called **rotation,** or *spin.* Both the carnival ride and the skater rotate.

When an object turns about an *external* axis, the motion is called **revolution.** Although the platform of the carnival ride rotates, the riders along the platform's outer edge *revolve* about its axis.

Earth undergoes both types of rotational motion. It revolves around the sun once every 365 $\frac{1}{4}$ days,* and it rotates around an axis passing through its geographical poles once every 24 hours.**

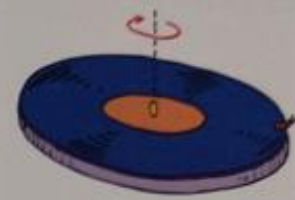

Figure 9.2 ▲
The turntable *rotates* around its axis while a ladybug sitting at its edge *revolves* around the same axis.

9.2 Rotational Speed

We began this chapter by asking which moves faster on a merry-go-round, a horse near the outside rail or one near the inside rail. Similarly, which part of a turntable moves faster? On the pre-CD record player shown in Figure 9.2, which part of the record moves faster under the stylus—the groove at the outer part of the record or the groove near the center? If you ask people these questions you'll probably get more than one answer, because some people will think about linear speed while others will think about rotational speed.

Linear speed, which we simply called speed in Chapter 2, is the distance moved per unit of time. A point on the outer edge of a merry-go-round or turntable moves a greater distance in one complete rotation than a point near the center. The linear speed is greater on the outer edge of a rotating object than it is closer to the axis. The speed of something moving along a circular path can be called **tangential speed** because the direction of motion is always tangent to the circle. For circular motion, we can use the terms linear speed and tangential speed interchangeably.

Rotational speed (sometimes called angular speed) is the number of rotations per unit of time. All parts of the rigid merry-go-round and the turntable rotate about their axis *in the same amount of time.* Thus, all parts have the same rate of rotation.

* Can you see the reason for leap years? Since Earth takes $\frac{1}{4}$ day more than 365 days to circle the sun, an extra day is added to the calendar every fourth year.

** With respect to the sun, Earth rotates once each 24-hour period. A 24-hour day is the time required for a point on Earth that is located directly under the sun to rotate and reach that point again. But with respect to the stars, a complete rotation of Earth takes 23 hours and 56 minutes. Why? Because while Earth rotates, it revolves about a degree around the sun in its orbit.

1 Explore | 2 Develop | 3 Apply

1 Laboratory Manual 30

or the same *number of rotations per unit of time*. It is common to express rotational speed in revolutions per minute (RPM).* For example, phonograph records that were common in the past rotate at $33\frac{1}{3}$ RPM. A ladybug sitting anywhere on the surface of the record revolves at $33\frac{1}{3}$ RPM.

Tangential speed and rotational speed are related. Have you ever ridden on a giant rotating platform in an amusement park? The faster it turns, the faster your tangential speed is. Tangential speed is directly proportional to the rotational speed and the radial distance from the axis of rotation. So we state**

Tangential speed ~ radial distance × rotational speed

At the center of the rotating platform, right at its axis, you have no tangential speed at all, but you do have rotational speed. You merely rotate. As you move away from the center, you move faster and faster—your tangential speed increases while your rotational speed stays the same. Move out twice as far from the center, and you have twice the tangential speed. Move out three times as far, and you have three times as much tangential speed. When a row of skaters locked arm in arm at the skating rink makes a turn, the motion of tail-end Charlie is evidence of this greater speed.

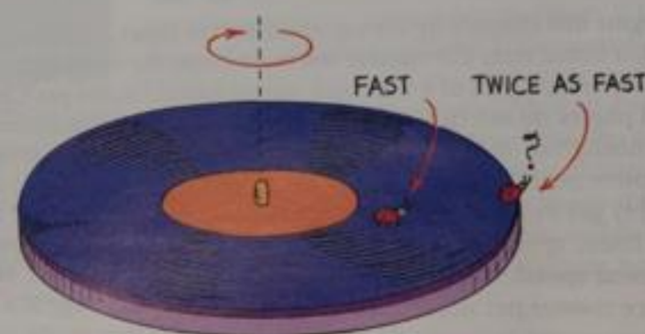

Figure 9.3 ▶
All parts of the turntable rotate at the same rotational speed, but ladybugs at different distances from the center travel at different linear speeds. A ladybug sitting twice as far from the center moves twice as fast.

To summarize: In any rigidly rotating system, all parts have the same rotational speed. However, the linear or tangential speed varies. Tangential speed depends on rotational speed and the distance from the axis of rotation.

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* Rotation rate is described as turns per unit of time, or as the angle turned (degrees or radians) per unit of time. The symbol for rotational speed is ω (the Greek letter omega). Examples of units for rotational speed are RPM, degrees per second, or radians per second. One degree is 1/360 of a full turn. One radian is about 1/6 of a full turn—precisely, it is $1/2\pi$ times 360 degrees, or 57.3 degrees. You may learn more about radians in a follow-up course.

** If you take a follow-up physics course you'll learn that when the proper units are used for tangential speed v, rotational speed ω, and radial distance r, the direct proportion of v to r and ω becomes the exact equation $v = r\omega$. This relationship applies only to a rotating system wherein all parts simultaneously have the same ω, like a rigid disk or rigid rod. It does not apply to a system of planets, for example, where each planet has a different rotational speed ω. (We will learn later that the innermost planets in a planetary system have both the greatest rotational speed *and* the greatest linear speed.)

DOING PHYSICS

Comparing Rolls

Roll a cylindrical can across a table. Note that the distance it rolls in each complete rotation equals the circumference of the can. Note also the can rolls in a straight-line path. Now roll an ordinary tapered drinking cup on the table (a paper or foam cup works fine). The wide end has a greater radius than the narrow end. Does the cup roll straight or does it curve? Does the wide end of the cup cover more distance as it rotates? Is the linear speed of the wide end greater? Doesn't this nicely show that linear speed depends on radius?

Activity

Questions

1. Which part of Earth's surface has the greatest rotational speed about Earth's axis? Which part has the greatest linear speed relative to Earth's axis?
2. On a particular merry-go-round, the horses along the outer rail are located three times farther from the axis of rotation than the horses along the inner rail. If a boy sitting on a horse near the inner rail has a rotational speed of 4 RPM and a tangential speed of 2 m/s, what will be the rotational speed and tangential speed of his sister who is sitting on a horse along the outer rail?
3. Trains ride on a pair of tracks. For straight-line motion, both tracks are the same length. But which track is longer for a curve, the one on the outside or the one on the inside of the curve?

Answers

1. Like a rotating turntable, all parts have the same rotational speed. The equatorial region has the greatest linear speed because it is farthest from the axis.
2. The rotational speed of the sister is also 4 RPM. Her tangential speed is 6 m/s. Since the merry-go-round is rigid, all the horses have the same rotational speed. But the horse at the outer rail is three times the distance from the center and thus has three times the tangential speed.
3. The outside track is longer, just as a circle with a greater radius has a longer circumference.

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LINK TO TECHNOLOGY

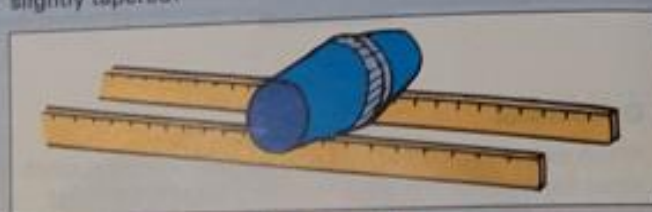
Railroad Wheels

When a train follows a curve, the outside wheels must travel faster than the inside wheels. Train wheels are tapered, and railroad tracks are slightly rounded so that only a small part of the wheel rides on the track at any time. When a train makes a left turn the tendency of the train to go straight puts the large-diameter part of the tapered right wheel on the right track and the small-diameter part of the tapered left wheel on the left track. Since both wheels, being attached to the same axis, have the same rotational speed, the right wheel has the greater linear speed required to make the turn.

DOING PHYSICS

Rolling on Tapered Wheels

Tape a pair of paper or foam cups together as shown. The pair doesn't roll well along a table, but rolls very well along a set of tracks. Set two meter sticks to simulate "railroad tracks." Set them parallel, about one cup length apart. Roll the cups on the tracks. When the cups are centered so parts of equal radius make contact with the tracks, straight-line motion results—both sides have the same linear speed. Roll the cups a bit off center and notice how they self-correct. Do you see that when a wider part on one cup rides the track, it moves faster than the narrow part of the other cup riding on the opposite track? This motion steers the cups toward the middle of the tracks. If the rolling cups "overshoot" the middle, does the same process occur on the other side to steer the cups back toward the middle? Do you think the wheels on a railroad car are cylindrical or slightly tapered?



Activity

Figure 9.4 ▲
Like the cup in the Doing Physics activity above, the rims of wheels on railroad tracks are tapered. Can you see when the wheels shift to the left of center, the left wheel rides on a larger diameter than the right wheel?

9.3 Centripetal Force

Whirl a tin can on the end of a string and you'll notice that you must keep continuously pulling on the string. You pull inward on the string to keep the can revolving over your head in a circular path. A force of some kind is required to maintain circular motion. Any force that causes an object to follow a circular path is called a **centripetal force.** *Centripetal* means "center-seeking," or "toward the center." The force that holds the occupants safely in the rotating carnival ride (Figure 9.1) is a center-directed force. Without it, the motion of the occupants would be along a straight line—they would not revolve.

Centripetal force is not a new kind of force. It is simply the name given to *any* force that is directed at a right angle to the path of a moving object and that tends to produce circular motion. Gravitational and electrical forces act across empty space as centripetal forces. Gravitational force directed toward the center of Earth holds the moon in an almost circular orbit around Earth. Electrons revolving around the nucleus of the atom are held in their orbits by an electrical force that is directed inward toward the nucleus.

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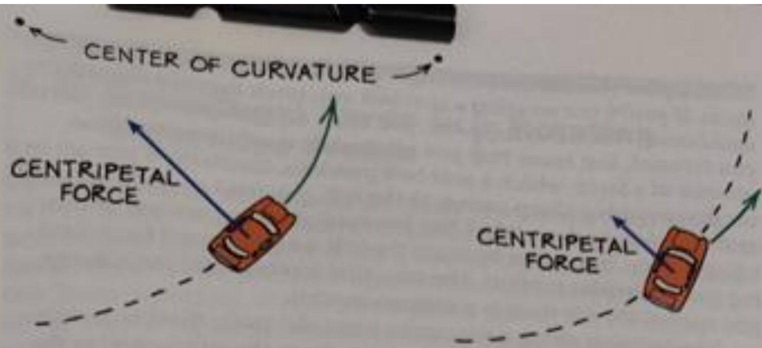

◄ **Figure 9.5**
(Left) For the car to go around a curve, there must be sufficient friction to provide the required centripetal force. (Right) If the force of friction is not great enough, skidding occurs.

When an automobile rounds a corner, the sideways-acting friction between the tires and the road provides the centripetal force that holds the car on a curved path (Figure 9.5). If the force of friction is not great enough, the car fails to make the curve and the tires slide sideways. The car skids.

Centripetal force plays the main role in the operation of a centrifuge. A familiar example is the spinning tub in an automatic washing machine. The tub rotates at high speed during its spin cycle. The tub's inner wall exerts a centripetal force on the wet clothes, which are forced into a circular path. The tub exerts great force on the clothes, but the holes in the tub prevent the tub from exerting the same force on the water in the clothes. The water escapes through the holes. It is important to note that a force acts on the clothes, not the water. It is not a force that causes the water to escape. The water escapes because it tends to move by inertia in a straight-line path (Newton's first law) *unless* acted on by a centripetal force or any other force. So interestingly enough, the clothes are forced away from the water, and not the other way around.

Figure 9.6 ▲
The clothes are forced into a circular path, but the water is not.

9.4 Centripetal and Centrifugal Forces

In the preceding examples, circular motion is described as being caused by a center-directed force. Sometimes an outward force is also attributed to circular motion. This outward force is called **centrifugal force.*** *Centrifugal* means "center-fleeing," or "away from the center." In the case of the whirling can, it is a common misconception to state that a centrifugal force pulls outward on the can. If the string holding the whirling can breaks (Figure 9.7), it is often wrongly stated that centrifugal force pulls the can from its circular path. But in fact, when the string breaks the can goes off in a tangential straight-line path because *no* force acts on it. We illustrate this further with another example.

Figure 9.7 ▲
When the string breaks, the whirling can moves in a straight line, tangent to—not outward from the center of—its circular path.

* Both centrifugal force and centripetal force depend on the mass m, tangential speed v, and radius of curvature r of the circularly moving object. If you take a follow-up physics course you'll learn the exact relationship $F = mv^2/r$.

Suppose you are the passenger in a car that suddenly stops short. If you're not wearing a seat belt you pitch forward toward the dashboard. When this happens, you don't say that something forced you forward. You know that you pitched forward because of the *absence* of a force, which a seat belt provides. Similarly, if you are in a car that rounds a sharp corner to the left, you tend to pitch outward against the right door. Why? Not because of some outward or centrifugal force, but rather because there is no centripetal force holding you in circular motion. The idea that a centrifugal force bangs you against the car door is a misconception.

So when you swing a tin can in a circular path, there is *no* force pulling the can outward. Only the force from the string acts on the can to pull the can inward. The outward force is *on the string*, not on the can.

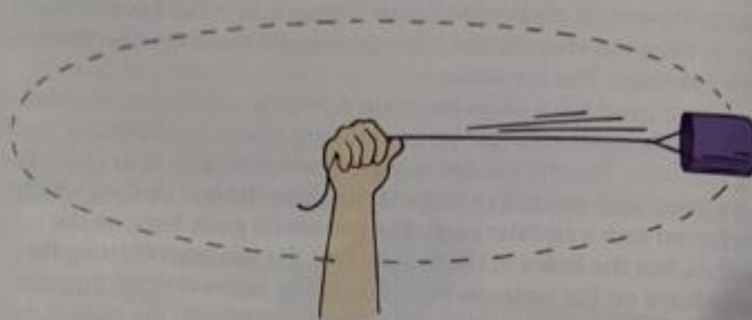

Figure 9.8 ▶
The only force that is exerted *on* the whirling can (neglecting gravity) is directed *toward* the center of circular motion. This is a *centripetal* force. No outward force acts on the can.

Now suppose there is a ladybug inside the whirling can (Figure 9.9). The can presses against the bug's feet and provides the centripetal force that holds it in a circular path. The ladybug in turn presses against the floor of the can. Neglecting gravity, the *only* force exerted *on the ladybug* is the force of the can on its feet. From our outside stationary frame of reference, we see there is no centrifugal force exerted on the ladybug, just as there is no centrifugal force exerted on the person who pitches against the car door. The "centrifugal-force effect" is attributed not to any real force but to inertia—the tendency of the moving body to follow a straight-line path. But try telling that to the ladybug!

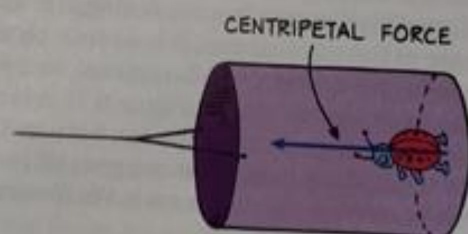

Figure 9.9 ▲
The can provides the centripetal force necessary to hold the ladybug in a circular path.

1 *Explore* **2** *Develop* **3** *Apply*

- 2 Concept-Development Practice Book *9-1*
- 3 Problem-Solving Exercises in Physics *6-1*

9.5 Centrifugal Force in a Rotating Reference Frame

Our view of nature depends on the frame of reference from which we view it. For instance, when sitting on a fast-moving train, we have no speed at all relative to the train, but we have an appreciable speed relative to the reference frame of the stationary ground outside. From one frame of reference we have speed; from another we have none—likewise with force. Recall the ladybug inside the whirling can. From a stationary frame of reference outside the whirling can, we see there is *no* centrifugal force acting on the ladybug inside the whirling can. However, we do see centripetal force acting on the can, producing circular motion. Thus, from an outside stationary frame of reference the *only* force acting on the ladybug is the centripetal force exerted by the bottom of the can on the ladybug's feet.

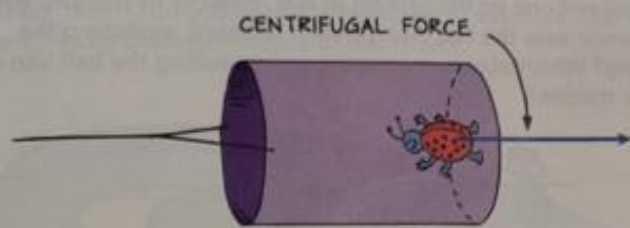

Figure 9.10 From the reference frame of the ladybug inside the whirling can, the ladybug is being held to the bottom of the can by a force that is directed away from the center of circular motion. The ladybug calls this outward force a *centrifugal* force, which is as real to the ladybug as gravity.

DOING PHYSICS

Water-Bucket Swing

Half fill a bucket of water and swing it in a vertical circle. If you swing it fast enough, the water won't fall out at the top. Interestingly enough, although it won't fall *out*, it still falls. The trick is to swing the bucket fast enough that the bucket falls as fast as the water inside falls. Can you see that because the bucket is revolving, the water moves tangentially as it falls—and stays in the bucket? Later we'll learn that an orbiting space shuttle similarly falls while in orbit. The trick is to give the shuttle sufficient tangential velocity that it falls around the curved Earth rather than into it.

Activity

PHYSICS ON THE JOB

Roller Coaster Designer

Since 1884, when the first American roller coaster was constructed, roller coasters have evolved into thrilling machines that rise over 100 meters high and reach speeds of over 150 km/h. Roller coaster designers, or mechanical design engineers, use the laws of physics to create rides that are both exciting and safe. In particular, designers must understand how roller coasters can safely navigate tall loops without exerting too much force on the riders. Designers of modern roller coasters first test their designs on computers to identify any problems before construction begins. Many private companies design roller coasters for amusement parks throughout the world.

But nature seen from the reference frame of the rotating system is different. In the rotating frame of reference of the whirling can, both centripetal force (supplied by the can) *and* centrifugal force act *on the ladybug.* The centrifugal force appears as a force in its own right, as real as the pull of gravity. However, there is a fundamental difference between the gravity-like centrifugal force and actual gravitational force. Gravitational force is always an interaction between one mass and another. The gravity we feel is due to the interaction between our mass and the mass of Earth. However, in a rotating reference frame the centrifugal force has no agent such as mass—there is no interaction counterpart. Centrifugal force is an effect of rotation. It is not part of an interaction and therefore it cannot be a true force. For this reason, physicists refer to centrifugal force as a

Questions

1. A heavy iron ball is attached by a spring to a rotating platform, as shown in the sketch. Two observers, one in the rotating frame and one on the ground at rest, observe its motion. Which observer sees the ball being pulled outward, stretching the spring? Which observer sees the spring pulling the ball into circular motion?

2. The spring in the sketch stretches 10 cm when the ball is located midway between the axis and the outer edge of the rotating circular platform. The spring support is then moved so the ball is located directly over the platform's outer edge, effectively doubling the ball's distance from the axis. Will the spring stretch more than, less than, or the same (10 cm) as it did before the support was moved?

Answers

1. The observer in the reference frame of the rotating platform states that centrifugal force pulls radially outward on the ball, which stretches the spring. The observer in the rest frame states that centripetal force supplied by the stretched spring pulls the ball into circular motion. (Only the observer in the rest frame can identify an action–reaction pair of forces; where action is spring-on-ball, reaction is ball-on-spring. The rotating observer can't identify a reaction counterpart to the centrifugal force because there isn't any.)
2. The ball has twice the linear speed when it's located at twice the distance from the rotational axis. With this greater speed there is a greater centripetal/centrifugal force. The spring therefore stretches more when the ball is near the edge. (We will see in Chapter 18 that the stretch of a spring is directly proportional to the applied force. This means that in this example, when the force is doubled, the stretch will double, from 10 cm to 20 cm.)

ors 2 Develop 3 Apply

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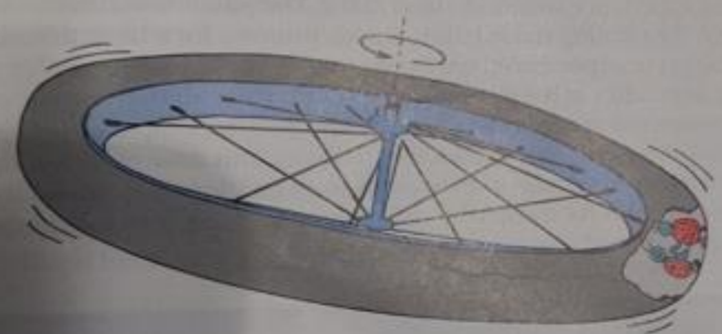
ctice Book 9-2

fictitious force, unlike gravitational, electromagnetic, and nuclear forces. Nevertheless, to observers who are in a rotating system, centrifugal force is very real. Just as gravity is ever present at Earth's surface, centrifugal force is ever present within a rotating system.

9.6 Simulated Gravity

Consider a colony of ladybugs living inside a bicycle tire—the balloon kind used on mountain bikes that has plenty of room inside. If we toss the wheel through the air or drop it from an airplane high in the sky, the ladybugs will be in a weightless condition. They will seem to float freely while the wheel is in free fall. Now spin the wheel. The ladybugs will feel themselves pressed to the outer part of the tire's inner surface. If the wheel is spun at just the right speed, the ladybugs will experience *simulated gravity* that feels like the gravity they are accustomed to. Gravity is simulated by centrifugal force. The "down" direction to the ladybugs is what we call "radially outward," away from the center of the wheel.



◀ **Figure 9.11**
If the spinning wheel freely falls, the ladybugs inside will experience a centrifugal force that feels like gravity when the wheel spins at the appropriate rate. To the occupants, the direction "up" is toward the center of the wheel and "down" is radially outward.

Today we live on the outer surface of our spherical planet, held here by gravity. Earth has been the cradle of humankind. But we will not stay in the cradle forever. We are on our way to becoming a spacefaring people. In the years ahead many people will likely live in huge lazily rotating space stations where centrifugal force simulates gravity. The simulated gravity will be provided so the people can function normally.

Occupants in today's space shuttles feel weightless because they lack a support force. They're not pressed against a supporting floor by gravity, nor do they experience a centrifugal force due to spinning. But future space travelers need not be subject to weightlessness. Their space habitats will probably spin, like the ladybug's spinning bicycle wheel, effectively supplying a support force and nicely simulating gravity.

The comfortable 1 *g* we experience at Earth's surface is due to gravity. Inside a rotating spaceship the acceleration experienced is the centripetal/centrifugal acceleration due to rotation. The magnitude of this acceleration is directly proportional to the radial distance and the square of the rotational speed. For a given RPM, the

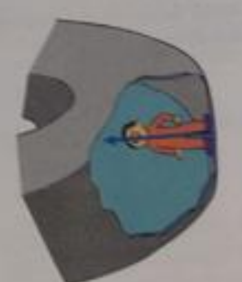


Figure 9.12 ▲
The interaction between the man and the floor as seen at rest outside the rotating system. The floor presses against the man (action) and the man presses back on the floor (reaction). The only force exerted on the man is by the floor. It is directed toward the center and is a centripetal force.

acceleration, like the linear speed, increases with increasing radial distance. Doubling the distance from the axis of rotation doubles the centripetal/centrifugal acceleration. Tripling the distance triples the acceleration, and so forth. At the axis where radial distance is zero, there is no acceleration due to rotation.

Small-diameter structures would have to rotate at high speeds to provide a simulated gravitational acceleration of 1 *g*. Sensitive and delicate organs in our inner ears sense rotation. Although there appears to be no difficulty at a single revolution per minute (1 RPM) or so, many people have difficulty adjusting to rotational rates greater than 2 or 3 RPM (although some people easily adapt to 10 or so RPM). To simulate normal Earth gravity at 1 RPM requires a large structure—one almost 2 km in diameter. This is an immense structure compared with the size of today's space shuttle vehicles. Economics will probably dictate that the size of the first inhabited structures be small. If these structures also do not rotate, the inhabitants will have to adjust to living in a seemingly weightless environment. Larger rotating habitats with simulated gravity will likely follow later.

If the structure rotates so that inhabitants on the inside of the outer edge experience 1 *g*, then halfway between the axis and the outer edge they would experience only 0.5 *g*. At the axis itself they would experience weightlessness at 0 *g*. The possible variations of *g* within the rotating space habitat holds promise for a most different and as yet unexperienced environment. We could perform ballet at 0.5 *g*; acrobatics at 0.2 *g* and lower *g* states; three-dimensional soccer and sports not yet conceived in very low *g* states. People will explore possibilities never before available to them. This time of transition from our earthly cradle to new vistas is an exciting time in which to live—especially for those who will be prepared to play a role in these new adventures.*

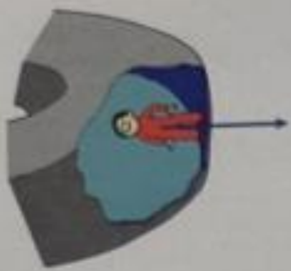


Figure 9.13 ▲
As seen from inside the rotating system, in addition to the man-floor interaction there is a centrifugal force exerted on the man at his center of mass. It seems as real as gravity. Yet, unlike gravity, it has no reaction counterpart—there is nothing out there that he can pull back on. Centrifugal force is not part of an interaction, but results from rotation. It is therefore called a fictitious force.

Figure 9.14 ▶
A NASA depiction of a rotational space colony.

1 *Explore* **2** *Develop* **3** *Apply*

2 Concept-Development Practice Book 9-3

* At the risk of stating the obvious, this preparation can well begin by taking your study of physics very seriously.

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9 Chapter Assessment

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Concept Summary

An object rotates when it turns around an internal axis; it revolves when it turns around an external axis.

- Rotational speed is the number of rotations or revolutions made per unit of time.

A centripetal force pulls objects toward a center.

- An object moving in a circle is acted on by a centripetal force.
- When an object moves in a circle, there is no force pushing the object outward from the circle.
- From within a rotating frame of reference, there seems to be an outwardly directed centrifugal force, which can simulate gravity.

Key Terms

axis (9.1)	revolution (9.1)
centrifugal force (9.4)	rotation (9.1)
centripetal force (9.3)	rotational speed (9.2)
linear speed (9.2)	tangential speed (9.2)

Review Questions *Check Concepts*

1. Distinguish between a rotation and a revolution. (9.1)
2. Does a child on a merry-go-round revolve or rotate around the merry-go-round's axis? (9.1)
3. Distinguish between linear speed and rotational speed. (9.2)
4. What is linear speed called when something moves in a circle? (9.2)
5. At a given distance from the axis, how does linear (or tangential) speed change as rotational speed changes? (9.2)
6. At a given rotational speed, how does linear (or tangential) speed change as the distance from the axis changes? (9.2)
7. When you roll a cylinder across a surface it follows a straight-line path. A tapered cup rolled on the same surface follows a circular path. Why? (9.2)
8. When you whirl a can at the end of a string in a circular path, what is the direction of the force that acts on the can? (9.3)
9. Does the force that holds the riders on the carnival ride in Figure 9.1 act toward or away from the center? (9.3)
10. Does an inward force or an outward force act on the clothes during the spin cycle of an automatic washer? (9.3)
11. When a car makes a turn, do seat belts provide you with a centripetal force or centrifugal force? (9.4)
12. If the string that holds a whirling can in its circular path breaks, what causes the can to move in a straight-line path—centripetal force, centrifugal force, or a lack of force? What law of physics supports your answer? (9.4)
13. Identify the action and reaction forces in the interaction between the ladybug and the whirling can in Figure 9.10. (9.5)
14. A ladybug in the bottom of a whirling tin can feels a centrifugal force pushing it against the bottom of the can. Is there an outside source of this force? Can you identify this as the action force of an action-reaction pair? If so, what is the reaction force? (9.5)
15. Why is the centrifugal force the ladybug feels in the rotating frame called a fictitious force? (9.5)

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Chapter Assessment 9

Review Questions

1. Rotation—about an axis within the body; revolution—around an axis external to the body
2. Revolve
3. Linear—distance/time; rotational—angle/time or number of rotations/time
4. Tangential speed
5. Directly, $v \sim r\omega$
6. Directly, $v \sim r\omega$
7. Outer diameter has a greater linear speed.
8. Inward, toward the center of the circle
9. Toward
10. Inward
11. Centripetal
12. Lack of a force; Newton's first law—inertia
13. Bug on can, can on bug
14. No; no; none
15. It is not part of an interaction.
16. Simulated gravitational force ~ ω^2
17. Simulated gravitational force ~ r
18. So they can simulate gravity at low rotational speeds