**Introduction**

**Purpose** Observe parabolic trajectory as part of projectile motion.

**Discussion**

Whenever motion is not strictly horizontal or vertical, vector components are needed to resolve angles, distances, and speed. For instance, to determine the vertical height that a projectile falls when a trajectory begins at an angle above horizontal (*as shown below*), one would need to find the vertical velocity component and use the equation: **dy = Vyt + ½ gt2**. To find the vertical velocity component, **Vy** on the diagram below, use **sin θ = Vy / Vi**. Therefore, **Vy = Vi sin θ**.

Likewise, to determine the horizontal distance that a projectile travels when a trajectory begins at an angle above horizontal (*as shown below*), one would need to find the horizontal velocity component and use the equation: **dx = Vxt + ½ gt2**. Since, there is no vertical component when dealing strictly with the horizontal component, gravity (g) is not a factor. Therefore, **dx = Vxt**.To find the horizontal velocity component, **Vy** on the diagram below, use **cos θ = Vx / Vi**. Therefore, **Vx = Vi cos θ**.

For the canon ball trajectory shown below, if we know the initial velocity component, Vi, we can calculate the horizontal and vertical component velocities of the canon using **cos θ** and **sin θ**.



**vi**

**θ**

**vx**

**Vy**

**Vy**

**Hypothesis**

If a projectile is launched from a non-horizontal position and off a cliff, then trajectory angle, velocity and distances can be calculated using trigonometry.

**Materials**

PHET Simulation <https://phet.colorado.edu/en/simulation/projectile-motion>

 You can access simulations at: <https://phet.colorado.edu/en/simulations/browse>

 Find the course you desire (Physics, Chemistry, Math, Earth Science, Biology). Go to the right to scroll each course for simulations.

**Procedures Part 1**

1. Click on the “Intro” simulation.



2. Set the initial speed to 12 m/s.

3. Choose the cannonball as the object to launch.

4. Use the target icon to measure the horizontal distances (dx).

5. Use the tape measure to measure the vertical distances (dy).

6. Complete the data table using the simulation.

|  |  |  |
| --- | --- | --- |
| Trajectory | Measured Horizontal Distancedx | Measured Vertical Distancedy |
| 15° |  |  |
| 30° |  |  |
| 45° |  |  |
| 60° |  |  |
| 75° |  |  |

Since we know the initial launch speed, Vi, we can calculate the horizontal vector components of speed, distance, and time as well as the vertical vector components.

Part 2 Horizontal Vector components

1. Copy the values of dx and dy from the previous table into the table below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Trajectory | Measured Horizontal Distancedx | **Calculated Horizontal Speed****Vx** | Measured Vertical Distancedy | **Calculated Time****t** | **Calculated dx** |
| 15° |  |  |  |  |  |
| 30° |  |  |  |  |  |
| 45° |  |  |  |  |  |
| 60° |  |  |  |  |  |
| 75° |  |  |  |  |  |

2. Calculate the horizontal launch velocity, Vx, for each angle of trajectory, using the equation: Vx = Vi cos θ. Add these values to the data table. Show Work for at least two speeds:

Vx = Vi cos 15° 🡪

Vx = Vi cos ?° 🡪

3. Using the vertical distance the object fell, dy, calculate the time it took for the object to fall at each angle of trajectory: dy = ½ gt2 rearranges to **t = √(2dy / g)**. Show Work for at least two times (t):

15° 🡪t = √(2dy / g) **=**

?° 🡪t = √(2dy / g) **=**

4. Using the measured horizontal distance the object travelled and the calculated time (from vertical distance fallen), calculate the horizontal distance, dx, using: **dx = Vx x t**. Show work for two distances:

15° 🡪dx = Vx x t **=**

?° 🡪dx = Vx x t **=**

Part 3 Vertical Vector components

1. Copy the values of the measured vertical distance, dy, from the data table in Part 1 and the time it took for the object to fall, t (calculated time to fall), from the data table in Part 2 into the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Trajectory | Measured Vertical Distancedy | **Calculated Vertical Speed****Vy** | Calculated Timet | **Calculated Vertical Distance****dy** |
| 15° |  |  |  |  |
| 30° |  |  |  |  |
| 45° |  |  |  |  |
| 60° |  |  |  |  |
| 75° |  |  |  |  |

2. Calculate the vertical launch velocity, Vy, for each angle of trajectory, using: Vy = Vi sin θ. Add these values to the data table. Show Work for at least two speeds:

Vy = Vi sin 15° 🡪

Vy = Vi sin ?° 🡪

3. Calculate the vertical distance the object fell, dy, at each angle of trajectory, using the equation: dy = Vyt + ½ gt2. Show Work for at least two distances:

15° 🡪 dy = Vyt + ½ gt2 =

?° 🡪 dy = Vyt + ½ gt2 =

**Conclusions and Questions**

1. Explain why only Vx = dx / t, and Vy = ½ gt2 are used in certain calculations rather than the overall displacement equation for projectiles: d = Vit + ½ gt2.

2. How do the horizontal component distances (measured versus calculated) compare? Likewise, how do the vertical component distances (measured versus calculated) compare? Explain any difference.

3. Draw a diagram showing the projectile path of the object through its entire flight for ONE angle of trajectory in this experiment. Then, draw a vector diagram on it, labeling the launch velocity, Vi, and the horizontal and vertical displacements (dx and dy).

Answers

**Procedures Part 1**



1. Click on the “Intro” simulation.

2. Set the initial speed to 12 m/s.

3. Choose the cannonball as the object to launch.

4. Use the target icon to measure the horizontal distances (dx).

5. Use the tape measure to measure the vertical distances (dy).

6. Complete the data table using the simulation.

|  |  |  |
| --- | --- | --- |
| Trajectory | Horizontal Distancedx | Vertical Distancedy |
| 15° | 20.5 m | 10.4 m |
| 30° | 22.4 m | 11.9 m |
| 45° | 21.5 m | 13.7 m |
| 60° | 17.0 m | 15.4 m |
| 75° | 9.4 m | 16.7 m |

Part 2 Horizontal Vector components

1. Copy the values of dx from the previous table into the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Trajectory | Measured Horizontal Distancedx | **Calculated Horizontal Speed****Vx** | **Calculated Time****t** | **Calculated dx** |
| 15° | 20.5 m | 11.6 m/s | 1.5 s | 17.4 m |
| 30° | 22.4 m | 10.4 m/s | 1.6 s | 16.6 m |
| 45° | 21.5 m | 8.5 m/s | 1.7 s | 14.5 m |
| 60° | 17.0 m | 6.0 m/s | 1.8 s | 10.8 m |
| 75° | 9.4 m | 3.1 m/s | 1.9 s | 5.9 m |

2. Calculate the horizontal launch velocity, Vx, for each angle of trajectory, using the equation: Vx = Vi cos θ. Add these values to the data table. Show Work for at least two speeds:

15° 🡪Vx  = Vi cos θ = (12 m/s)(cos 15°) = 11.6 m/s

30° 🡪Vx  = Vi cos θ = (12 m/s)(cos 30°) = 10.4 m/s

45° 🡪Vx  = Vi cos θ = (12 m/s)(cos 45°) = 8.5 m/s

60° 🡪Vx  = Vi cos θ = (12 m/s)(cos 60°) = 6.0 m/s

75° 🡪Vx  = Vi cos θ = (12 m/s)(cos 75°) = 3.1 m/s

3. Using the vertical distance the object fell, dy, calculate the time it took for the object to fall at each angle of trajectory. dy = ½ gt2 rearranges to **t = √(2dy / g)**. Show Work for at least two times:

15° 🡪t = √(2dy / g) **=** √(2 x 10.4 m / 9.8 m/s2) = 1.5 s

30° 🡪t = √(2dy / g) **=** √(2 x 11.9 m / 9.8 m/s2) = 1.6 s

45° 🡪t = √(2dy / g) **=** √(2 x 13.7 m / 9.8 m/s2) = 1.7 s

60° 🡪t = √(2dy / g) **=** √(2 x 15.4 m / 9.8 m/s2) = 1.8 s

75° 🡪t = √(2dy / g) **=** √(2 x 16.7 m / 9.8 m/s2) = 1.9 s

4. Using the measured horizontal distance the object travelled and the calculated time (from vertical distance fallen), calculate the horizontal distance, dx, using: **dx = Vx x t**. Show work for two distances:

15° 🡪dx = Vx x t **=** 11.6 m/s x 1.5 s **=** 17.4 m

30° 🡪dx = Vx x t= 10.4 x 1.6 s = 16.6 m

45° 🡪dx = Vx x t **=** 8.5 m/s x 1.7 s **=** 14.5 m

60° 🡪dx = Vx x t= 6.0 x 1.8 s = 10.8 m

75° 🡪dx = Vx x t **=** 3.1 m/s x 1.9 s **=** 5.9 m

Part 3 Vertical Vector components

1. Copy the values of dy from the data table in Part 1 into the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Trajectory | Measured Vertical Distancedy | **Calculated Vertical Speed****Vy** | Calculated Timet | **Calculated dy** |
| 15° | 10.4 m | 3.1 m/s | 1.5 s | 15.7 m |
| 30° | 11.9 m | 6.0 m/s | 1.6 s | 22.1 m |
| 45° | 13.7 m | 8.5 m/s | 1.7 s | 28.6 m |
| 60° | 15.4 m | 10.4 m/s | 1.8 s | 34.6 m |
| 75° | 16.7 m | 11.6 m/s | 1.9 s | 39.7 m |

2. Calculate the vertical launch velocity, Vy, for each angle of trajectory, using: Vy = Vi sin θ. Add these values to the data table. Show Work for at least two speeds:

15° 🡪Vy = Vi sin 15° = (12 m/s)(sin 15°) = 3.1 m/s

30° 🡪Vy = Vi sin 30° = (12 m/s)(sin 30°) = 6.0 m/s

45° 🡪Vy = Vi sin 45° = (12 m/s)(sin 45°) = 8.5 m/s

60° 🡪Vy = Vi sin 60° = (12 m/s)(sin 60°) = 10.4 m/s

75° 🡪Vy = Vi sin 75° = (12 m/s)(sin 75°) = 11.6 m/s

3. Calculate the vertical distance the object fell, dy, at each angle of trajectory, using the equation: dy = Vyt + ½ gt2. Show Work for at least two distances:

15° 🡪 dy = Vyt + ½ gt2 = (3.1 m/s x 1.5 s) + ½ (9.8 m/s2) x (1.5 s)2 = 15.7 m

30° 🡪 dy = Vyt + ½ gt2 = (6.0 m/s x 1.6 s) + ½ (9.8 m/s2) x (1.6 s)2 = 22.1 m

45° 🡪 dy = Vyt + ½ gt2 = (8.5 m/s x 1.7 s) + ½ (9.8 m/s2) x (1.7 s)2 = 28.6 m

60° 🡪 dy = Vyt + ½ gt2 = (10.4 m/s x 1.8 s) + ½ (9.8 m/s2) x (1.8 s)2 = 34.6 m

75° 🡪 dy = Vyt + ½ gt2 = (11.6 m/s x 1.9 s) + ½ (9.8 m/s2) x (1.9 s)2 = 39.7 m

**Conclusions and Questions**

1. Explain why we only use Vx = dx / t, and Vy = ½ gt2 for many measurements rather than the overall displacement equation for projectiles: d = Vit + ½ gt2.

*The horizontal component does not include gravity. The horizontal component has no gravity component so the ½ gt2 becomes 0. Therefore,* ***dx = vxt***

*The total vertical component requires a complicated calculation because it includes not only the free fall from the cliff height, but the amount of height above the cliff. The vertical component does not include Vit (when shot horizontally), but does when launched at an angle and when object are projected off a cliff.*

2. How do the horizontal component distances (measured versus calculated) compare? Likewise, how do the vertical component distances (measured versus calculated) compare? Explain any difference.

*The calculated distances (for both horizontal and vertical components) were greater in magnitude than the measured distances. However, the sequence was the same. For example, the vertical distance components increased from 15*° *to 75*°*. The object is launched upward so that initial distance must be included as part of the vertical distance. The highest point above the ground is what was used for dy initially.*

3. Draw a diagram showing the projectile path of the object through its entire flight for ONE angle of trajectory in this experiment. Then, draw a vector diagram on it, labeling the launch velocity, Vi, and the horizontal and vertical displacements (dx and dy).

**vi**

**θ**

**vx**

**Vy**

**Vy**

**dy**

**dx**